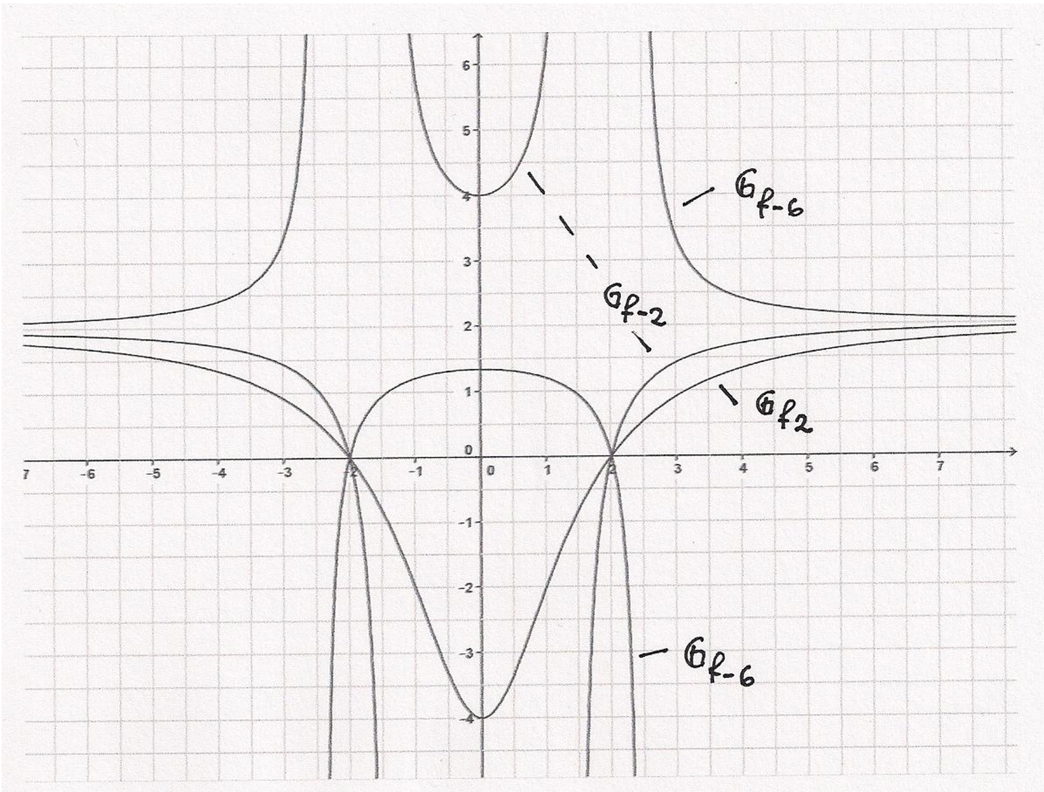
$f_k: \ x \mapsto \dfrac{2x^2 \ - 8}{x^2 + \ k}, k \ \in \ \mathbb{R} \setminus \{-4\}$

1. Definitionsmenge: $x^2 + \ k \ = 0; \ x^2 \ = \ -k.$
   $k > 0$: Keine Lösung; $\mathbb{D} \ = \ \mathbb{R}.$
   $k = 0: \mathbb{D} \ = \ \mathbb{R} \setminus \{0\}$
   $k < 0 \ (\neq -4): \mathbb{D} \ = \ \mathbb{R} \setminus \{\pm\sqrt{-k}\}$
   $f(0) \ = \ -\frac{8}{k}$; Schnittpunkt $(0| -\frac{8}{k})$
   $f_k(x) = \ 0; 2x^2 \ - 8 = 0; \ x^2 = 4; x = \ \pm 2;$ Schnittpunkte $(-2|0); (2|0)$
2. $f_k(-x) \ = \ f_k(x)$, Graph achsensymmetrisch zur y –Achse.
3. $\lim_{x\to\infty} f_k(x) \ = \ \lim\limits_{x\to\infty} \frac{x^2(\,2-\frac{8}{x^2})}{x^2\left(1+\frac{k}{x^2}\right)} \ = \ \lim\limits_{x\to\infty} \frac{2-\frac{8}{x^2}}{\left(1+\frac{k}{x^2}\right)} \ = \ 2 \ = \lim_{x\to-\infty} f_k(x)$; waagrechte Asymptote $y = 2.$
4. $k > 0$: Keine senkrechte Asymptote
   $k = 0: x = \ 0$ senkrechte Asymptote
   $k < 0 \ \ (\neq -4): x = \ \pm\sqrt{-k}$ senkrechte Asymptoten
5. $f_k'(x) = \ \frac{4x\,(x^2+\ k) - (2x^2\,-8\,)\,2x}{(x^2+\ k)^2} \ = \ \frac{4x^3+4kx-4x^3+\,16x}{(x^2+\ k)^2} \ = \ \frac{4x\,(k+4)}{(x^2+\ k)^2} \ = \ 0; x = 0.$
   $k > -4: x < 0: f_k'(x) < 0; \ \ x > 0: f_k'(x) > 0; f \ sma \to smz;$ TIP$(0| -\frac{8}{k})$;
   $k < -4: x < 0: f_k'(x) > 0; \ \ x > 0: f_k'(x) < 0; f \ smz \to sma;$ HOP$(0| -\frac{8}{k})$;
6. 

7. $k = -4: f_{-4}(x) \ = \ \frac{2x^2\,-8}{x^2-\ 4} \ = \ \frac{2(x^2\,-4)}{x^2-\ 4} = 2.$ Es handelt sich um eine lineare Funktion: $f_{-4}(x) \ = 2, \mathbb{D} \ = \ \mathbb{R} \setminus \{\pm 2\};$