Bestimme jeweils die Ableitungsfunktion – Lösung

a) $f(x)=\dfrac{-x^2+3x-1}{e^x}$, $f'(x)=\dfrac{e^x(-2x+3)-(-x^2+3x-1)e^x}{e^{2x}}$

b) $f(x)=x^{\frac{1}{3}}\cdot(5x-7)$, $f'(x)=\dfrac{1}{3}x^{-\frac{2}{3}}\cdot(5x-7)+x^{\frac{1}{3}}\cdot 5$

c) $f(x)=\ln(\dfrac{1}{3}x-a)$, $f'(x)=\dfrac{1}{\frac{1}{3}x-a}\cdot\dfrac{1}{3}$

d) $f(x)=\cos(3(x-e^x)^2)$, $f'(x)=-\sin(3(x-e^x)^2)\cdot 6\cdot(x-e^x)\cdot(1-e^x)$

e) $f(x)=5\cdot e^{x^2+a}\cdot\ln(\pi\cdot x)$, $f'(x)=5e^{x^2+a}\cdot 2x\cdot\ln(\pi x)+5e^{x^2+a}\cdot\dfrac{1}{\pi x}\cdot\pi$

f) $f(x)=e^{x^2-3}\cdot x^{\frac{4}{5}}$, $f'(x)=e^{x^2-3}\cdot 2x\cdot x^{\frac{4}{5}}+e^{x^2-3}\cdot\dfrac{4}{5}\cdot x^{-\frac{1}{5}}$

g) $f(x)=\ln(e^{x+2})\cdot\sin(x)$, $f'(x)=\dfrac{1}{e^{x+2}}\cdot e^{x+2}\cdot\sin(x)+\ln(e^{x+2})\cdot\cos(x)$

h) $f(x)=\sqrt[3]{x}\cdot e^{4x^2}$, $f'(x)=\dfrac{1}{3}x^{-\frac{2}{3}}\cdot e^{4x^2}+\sqrt[3]{x}\cdot e^{4x^2}\cdot 8x$

i) $f(x)=\dfrac{5x-3}{\ln(x^2-0{,}5)}$, $f'(x)=\dfrac{\ln(x^2-0{,}5)\cdot 5-(5x-3)\dfrac{1}{x^2-0{,}5}\cdot 2x}{(\ln(x^2-0{,}5))^2}$

j) $f(x)=\sin(e^{x-3})\cdot 2x^{\frac{3}{4}}$, $f'(x)=\cos(e^{x-3})\cdot e^{x-3}\cdot 2x^{\frac{3}{4}}+\sin(e^{x-3})\cdot 2\cdot\dfrac{3}{4}x^{-\frac{1}{4}}$

Leite ab und vereinfache so weit wie möglich - Lösung

a) $f(x) = \frac{\sqrt{2x}}{e^{3x+2}}$,

$$f'(x) = \frac{e^{3x+2}\,\frac{1}{2}(2x)^{-\frac{1}{2}} \cdot 2 - \sqrt{2x} \cdot e^{3x+2} \cdot 3}{(e^{3x+2})^2} = \frac{(2x)^{-\frac{1}{2}} - 3\sqrt{2x}}{e^{3x+2}} = \frac{\frac{1}{\sqrt{2x}} - \frac{3\sqrt{2x} \cdot \sqrt{2x}}{\sqrt{2x}}}{e^{3x+2}} = \frac{1-6x}{\sqrt{2x} \cdot e^{3x+2}}$$

...jetzt kann man die Nullstelle der Ableitung gut ablesen!

b) $f(x) = \frac{\cos x}{2 \cdot \sin x}$, $f'(x) = \frac{2\sin x \cdot (-\sin x) - \cos x \cdot 2\cos x}{(2 \cdot \sin x)^2} = \frac{-2(\sin^2 x + \cos^2 x)}{4\sin^2 x} = \frac{-1}{2\sin^2 x}$