

Exponentialgleichungen – Lösungen

1. $7^{3x-2} = 7^2 \rightarrow x = \frac{4}{3}$

2. $3^{2x-1} = 3^{1,5} \rightarrow x = \frac{5}{4}$

3. $8^{x+1} = 0,125$
 $2^{3x+3} = \frac{1}{2^3} \rightarrow x = -2$

4. $4^x + 4^{x+3} = 16,25$
 $4^x + 64 \cdot 4^x = 16,25$
 $4^x \cdot 65 = 16,25$
 $4^x = \frac{1}{4} \rightarrow x = -1$

5. $\frac{11^x}{11} + 1330 = 121 \cdot 11^x \quad | \cdot 11$
 $11^x + 14630 = 1331 \cdot 11^x$
 $14630 = 1330 \cdot 11^x$
 $11 = 11^x \rightarrow x = 1$

6. $5^{12x-2} + 5^{12x-3} - 5^{12x-4} = 145$
 $5^{12x-4} \cdot (5^2 + 5 - 1) = 145$
 $5^{12x-4} \cdot 29 = 145$
 $5^{12x-4} = 5$
 $12x - 4 = 1 \rightarrow x = \frac{5}{12}$

7. $4^x + \frac{4^2}{4^x} = 10 \quad | \cdot 4^x$
 $(4^x)^2 + 16 = 10 \cdot 4^x$
 $(4^x)^2 - 10 \cdot 4^x + 16 = 0$
 Substitution: $4^x = u$
 $u^2 - 10u + 16 = 0 \quad u_1 = 8, \quad u_2 = 2$
 Resubstitution: $4^x = 8 \rightarrow x_1 = \frac{3}{2}$
 $4^x = 2 \rightarrow x_2 = \frac{1}{2}$

8. Substitution: $3^x = u$

$$u^2 + 9 = 10u$$

$$u^2 - 10u + 9 = 0 \quad u_1 = 9, \quad u_2 = 1$$

$$\text{Resubstitution: } 3^x = 9 \rightarrow x_1 = 2$$

$$3^x = 1 \rightarrow x_2 = 0$$

9. $5^x = \frac{25}{5^x} + 24 \quad | \cdot 5^x$

$$(5^x)^2 = 25 + 24 \cdot 5^x$$

$$(5^x)^2 - 24 \cdot 5^x - 25 = 0$$

$$\text{Substitution: } 5^x = u$$

$$u^2 - 24u - 25 = 0 \quad u_1 = 25, \quad u_2 = -1$$

$$\text{Resubstitution: } 5^x = 25 \rightarrow x_1 = 2$$

$$5^x = -1 \rightarrow \text{geht nicht!}$$

10. $2 \cdot 3^x = 0,5 \cdot 2^{-x} \quad | \lg(\dots)$

$$\lg 2 + x \cdot \lg 3 = \lg 0,5 - x \cdot \lg 2$$

$$x \cdot \lg 3 + x \cdot \lg 2 = \lg 0,5 - \lg 2$$

$$x = \frac{\lg 0,5 - \lg 2}{\lg 3 + \lg 2} \rightarrow x \approx -0,774, \quad y \approx 0,855$$

11. $0,5 \cdot 4^x = 3 \cdot 0,75^x \quad | \lg(\dots)$

$$\lg 0,5 + x \cdot \lg 4 = \lg 3 + x \cdot \lg 0,75$$

$$x \cdot \lg 4 - x \cdot \lg 0,75 = \lg 3 - \lg 0,5$$

$$x = \frac{\lg 3 - \lg 0,5}{\lg 4 - \lg 0,75} \rightarrow x \approx 1,070, \quad y \approx 2,204$$

12. $49^x - 50 \cdot 25^{x-1} = 3 \cdot 7^{2x} - 4 \cdot 5^{2x}$

$$49^x - 2 \cdot 25^x = 3 \cdot 49^x - 4 \cdot 25^x$$

$$2 \cdot 25^x = 2 \cdot 49^x \rightarrow x = 0$$

13. $24 \cdot 2^{2x-2} - 4 \cdot 3^{2x} = 4^{x+1} - 9^x$

$$6 \cdot 2^{2x} - 4 \cdot 3^{2x} = 4 \cdot 4^x - 3^{2x}$$

$$6 \cdot 2^{2x} - 4 \cdot 2^{2x} = 4 \cdot 3^{2x} - 3^{2x}$$

$$2 \cdot 2^{2x} = 3 \cdot 3^{2x} \quad | \lg(\dots)$$

$$\lg 2 + 2x \cdot \lg 2 = \lg 3 + 2x \cdot \lg 3$$

$$2x \cdot (\lg 2 - \lg 3) = \lg 3 - \lg 2 \rightarrow x = -0,5$$