Sinus und Kosinus am Dreieck– Lösung

1.

a) $\beta = 180° - 90° - 60° = 30°$

$h_c = b \cdot \cos(\alpha) = \frac{5\sqrt{3}}{2}$

$\gamma_1 = \beta = 30° \;\; ; \;\; \gamma_2 = \alpha = 60°$

$\frac{b}{c} = \cos(\alpha) \rightarrow \;\; c = \frac{b}{\cos(\alpha)} = 10cm$

$\frac{a}{c} = \cos(\beta) \rightarrow a = c \cdot \cos(\beta) = 5\sqrt{3}\,cm$

Probe mit Pythagoras:

$a^2 + b^2 = c^2 \;\; \rightarrow \;\; \left(5\sqrt{3}\,cm\right)^2 + (5\,cm)^2 = 100 = 10^2$

b) $\frac{h_c}{a} = \sin(\beta) \rightarrow \;\; \beta = \sin^{-1}(\frac{h_c}{a}) = 75°$

$\gamma_1 = \beta = 75° \;\; ; \;\; \gamma_2 = \alpha = 15°$

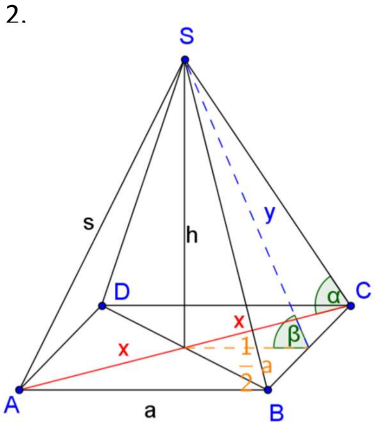
$\cos(\beta) = \frac{a}{c} \rightarrow c = \frac{a}{\cos(\beta)} = 16\,cm$

$\sin(\alpha) = \frac{h_c}{b} \;\; \rightarrow \;\; b = \frac{h_c}{\sin(\alpha)} = 4\sqrt{6} + 4\sqrt{2}\,cm$

Probe mit Pythagoras:

$a^2 + b^2 = c^2 \;\; \rightarrow \;\; \left(4\sqrt{6} - 4\sqrt{2}\,cm\right)^2 + \left(4\sqrt{6} + 4\sqrt{2}\right)^2 = 256 = 16^2$

2.



$a^2 + a^2 = (2x)^2 \;\; \rightarrow \;\; 32 = 4x^2 \;\; \rightarrow \;\; x = \sqrt{8}$

$\cos(\alpha) = \frac{x}{s} = \frac{\sqrt{8}}{6} \;\; \rightarrow \;\; \alpha = \cos^{-1}(\frac{\sqrt{8}}{6}) \approx 61{,}9°$

$$h^2 + x^2 = s^2$$

$$h^2 = 6^2 - \sqrt{8}^2 \;\; \rightarrow \;\; h = \sqrt{28} = 2\sqrt{7}$$

$y^2 = h^2 + \left(\frac{1}{2}a\right)^2 = 32 \;\; \rightarrow \;\; y = \sqrt{32} = 4\sqrt{2}$

$\cos(\beta) = \frac{\frac{1}{2}a}{y} \;\; \rightarrow \;\; \beta = \cos^{-1}(\frac{2}{4\sqrt{2}}) \approx 69{,}3°$

Verwendet man den Tangens, dann muss man y nicht berechnen.

$$\tan(\beta) = \frac{h}{\frac{1}{2}a} \;\; \rightarrow \;\; \beta = \tan^{-1}\left(\frac{2\sqrt{7}}{2}\right) \approx 69{,}3°$$

3. $\frac{\sin(40°)}{\sin(\beta)} = 1{,}5 \;\; \rightarrow \;\; \frac{\sin(40°)}{1{,}5} = \sin(\beta) \;\; \rightarrow \;\; \beta = \sin^{-1}\left(\frac{\sin(40°)}{1{,}5}\right) \approx 25{,}4°$

$$\sin(\beta) = \frac{2\,cm}{x} \;\; \rightarrow \;\; x = \frac{2\,cm}{\sin(\beta)} \approx 4{,}67\,cm$$