

Sinus und Kosinus am Dreieck– Lösung

1.

a) $\beta = 180^\circ - 90^\circ - 60^\circ = 30^\circ$

$$h_c = b \cdot \cos(\alpha) = \frac{5\sqrt{3}}{2}$$

$$\gamma_1 = \beta = 30^\circ ; \gamma_2 = \alpha = 60^\circ$$

$$\frac{b}{c} = \cos(\alpha) \rightarrow c = \frac{b}{\cos(\alpha)} = 10 \text{ cm}$$

$$\frac{a}{c} = \cos(\beta) \rightarrow a = c \cdot \cos(\beta) = 5\sqrt{3} \text{ cm}$$

Probe mit Pythagoras:

$$a^2 + b^2 = c^2 \rightarrow (5\sqrt{3} \text{ cm})^2 + (5 \text{ cm})^2 = 100 = 10^2$$

b) $\frac{h_c}{a} = \sin(\beta) \rightarrow \beta = \sin^{-1}\left(\frac{h_c}{a}\right) = 75^\circ$

$$\gamma_1 = \beta = 75^\circ ; \gamma_2 = \alpha = 15^\circ$$

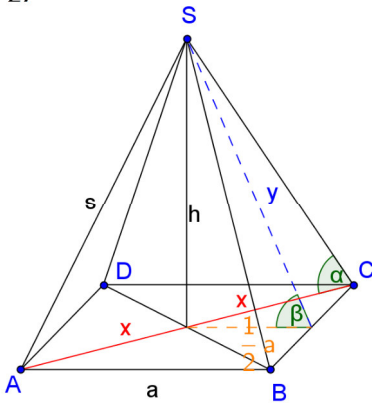
$$\cos(\beta) = \frac{a}{c} \rightarrow c = \frac{a}{\cos(\beta)} = 16 \text{ cm}$$

$$\sin(\alpha) = \frac{h_c}{b} \rightarrow b = \frac{h_c}{\sin(\alpha)} = 4\sqrt{6} + 4\sqrt{2} \text{ cm}$$

Probe mit Pythagoras:

$$a^2 + b^2 = c^2 \rightarrow (4\sqrt{6} - 4\sqrt{2} \text{ cm})^2 + (4\sqrt{6} + 4\sqrt{2})^2 = 256 = 16^2$$

2.



$$a^2 + a^2 = (2x)^2 \rightarrow 32 = 4x^2 \rightarrow x = \sqrt{8}$$

$$\cos(\alpha) = \frac{x}{s} = \frac{\sqrt{8}}{6} \rightarrow \alpha = \cos^{-1}\left(\frac{\sqrt{8}}{6}\right) \approx 61,9^\circ$$

$$h^2 + x^2 = s^2$$

$$h^2 = 6^2 - \sqrt{8}^2 \rightarrow h = \sqrt{28} = 2\sqrt{7}$$

$$y^2 = h^2 + \left(\frac{1}{2}a\right)^2 = 32 \rightarrow y = \sqrt{32} = 4\sqrt{2}$$

$$\cos(\beta) = \frac{\frac{1}{2}a}{y} \rightarrow \beta = \cos^{-1}\left(\frac{2}{4\sqrt{2}}\right) \approx 69,3^\circ$$

Verwendet man den Tangens, dann muss man y nicht berechnen.

$$\tan(\beta) = \frac{h}{\frac{1}{2}a} \rightarrow \beta = \tan^{-1}\left(\frac{2\sqrt{7}}{2}\right) \approx 69,3^\circ$$

3. $\frac{\sin(40^\circ)}{\sin(\beta)} = 1,5 \rightarrow \frac{\sin(40^\circ)}{1,5} = \sin(\beta) \rightarrow \beta = \sin^{-1}\left(\frac{\sin(40^\circ)}{1,5}\right) \approx 25,4^\circ$

$$\sin(\beta) = \frac{2 \text{ cm}}{x} \rightarrow x = \frac{2 \text{ cm}}{\sin(\beta)} \approx 4,67 \text{ cm}$$